Orbit reconstruction for GPS outages

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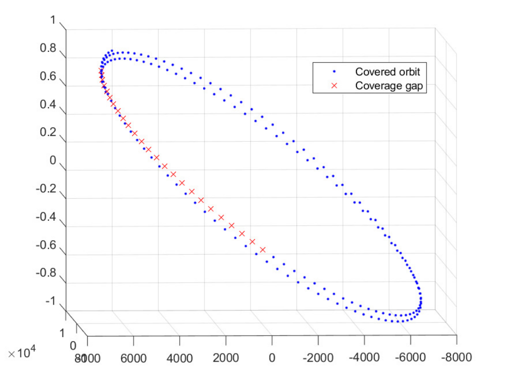
Our contribution

The scientific goal for timing resolution requires precision of 30 m.

The present algorithm is aimed at finding the satellite's position within *long* GPS coverage gaps, i. e. up to 25 min.



Corrections approach

We approximate the points inside the outage by

$$\mathbf{X}_{\mathrm{out}} \approx \mathbf{X}_{\mathrm{Kep.}} + \delta_1\mathbf{X} + \delta_2\mathbf{X}$$

Here

- $\mathbf{X}_{\mathrm{Kep.}}$ is Keplerian elliptical orbit with the J_2-correction for Earth's oblateness.
- $\delta_1\mathbf{X}$ first-order correction, uses points just before and after the gap.
- $\delta_2\mathbf{X}$ second-order correction: points lying 1 full quasi-period before and after the gap.

Keplerian orbit

The Keplerian orbit is constructed by utilising the two-body gravitational potential with J_2 harmonics as correction to it:

$$U = -\frac{GM}{r} + \frac{J_2(3\sin^2\theta - 1)}{2r^3}. \tag{1}$$

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- The orbit is constructed using Keplerian equations and the least-square method with 5 points before the gap as input. (See Vallado, *Fund. of Astrod. and Appl.*, 2013).
- After construction the orbit is propagated in the outage region to obtain $\mathbf{X}_{\mathrm{Kep.}}$.

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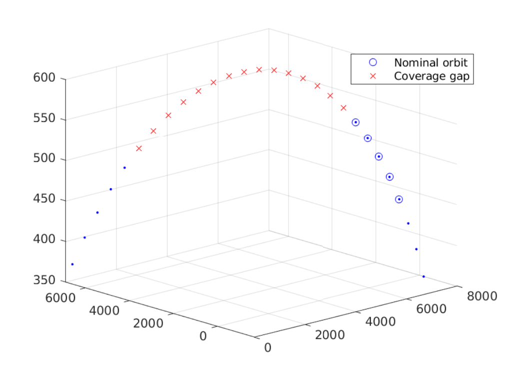


First-order correction

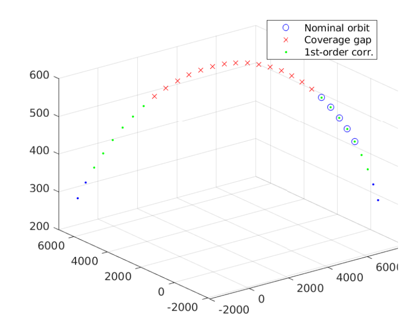
To find the first-order correction we

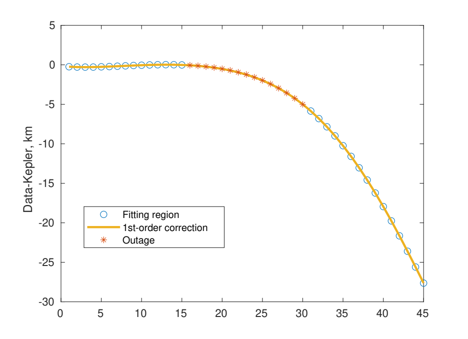
- take ca. 30 points lying before and 30 after the outage and compute for them the difference:

$$\mathbf{X} - \mathbf{X}_{\mathrm{Kep.}}$$

- Fit the difference by 8th—12th order polynomial for each coordinate.
- Interpolate the region in between. The result is $\delta_1\mathbf{X}$.
- For the data with $\sigma_{pos} = 3$ m, the probability to stay within 30 m is ca. 80 %.

First-order correction





Second-order correction

For the 2nd-order correction we use data from one Keplerian period before and after the outage:

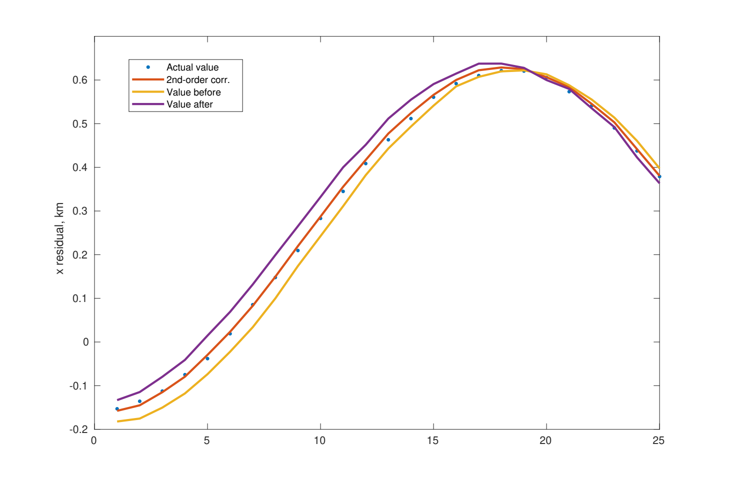
- Take points lying at $t_{\pm} = t_{\mathrm{out}} \pm T_{\mathrm{Kep.}}$ and compute nominal orbit + 1st order correction for them.
- For pretending outages starting at $t_{\pm}$ compute difference from the data:

$$\Delta \mathbf{X}_{\mathrm{b,a}} = \mathbf{X}_{\mathrm{b,a}} - \mathbf{X}_{\mathrm{Kep.}} - \delta_1 \mathbf{X}.$$

- Find the 2nd-order correction as the average value between "before" and "after"

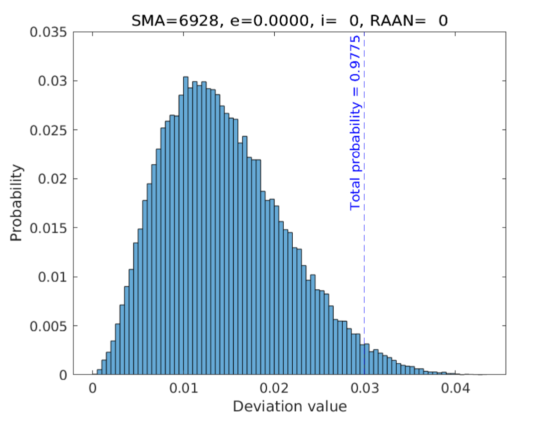
$$\delta_2 \mathbf{X} = \frac{\Delta \mathbf{X}_{\mathbf{b}} + \Delta \mathbf{X}_{\mathbf{a}}}{2}. \tag{1}$$

Second-order correction



Results

For the simulated fiducial HERMES orbit (LEEO, $e = 0$, $i = 0°$) with Gaussian noise the probability to stay within 30 m for a 25-minute long outage: $\approx$ 98%.



Additional analysis

In view of the future growth of the mission to HERMES-FC we investigated the performance of the algorithm for the orbits deviating from LEEO in either inclination or eccentricity.

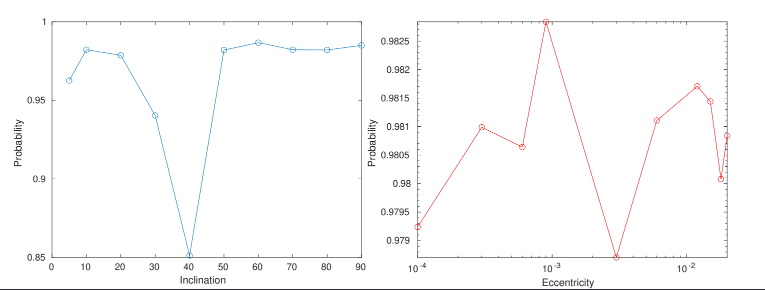


Figure: Probability dependence on inclination ($e = 10^{-4}$) and eccentricity ($i = 3°$).

Conclusions

- The algorithm works well for LEEO: Probability to stay within 30 m is $\approx$ 98%.
- The probability decreases with inclination and has minimum at $i \approx 40°$.
- No apparent dependence on eccentricity for low inclinations.
- The resulting probabilities are acceptable for the HERMES scientific needs.